Algebra

Second Packet with online option

Students and Parents,

I have listed the days and topics in the following chart with both a worksheet and online option. The worksheets will be posted with this on google classroom and it will also be available at the middle school office. The online assignments this time are through Clever at clever.com through the link to "Woot Math". Some online options in bold are from khanacademy. Video resources for these topics can be found on connected.mcgrawhill.com and there is also a link to it through Clever as well.

When you finish a worksheet you may scan it or take a picture of it and email it to me so I Can grade it.

Woot Math Class Code: 7LL9NZ and then click to sign in with Google to use your school gmail account

Choose one option below for each day

Day	Topic from packet	Online Option- Clever/Khanacademy Log into Clever and select Woot Math or Khanacademy for easy login	Worksheet Option
4/13	Square Root Functions	*Woot Math: "Graphing Cube root, Square root and Quadratic functions"*	Chapter 10: Lesson 1 Study Guide and Intervention- EVENS both sides ***I made a video of examples to help explain this lesson. It's posted to Google Classroom and Remind.***
4/14	Simplifying Radical Expressions	***Khanacademy: Simplify Square Root Expressions*** ***Video assigned to help***	*Chapter 10: Lesson 2 Study Guide and Intervention- EVENS both sides* ***See video examples I uploaded to help with this lesson***
4/15	Simplifying Radical Expressions	*Woot Math: Square Root Method"*	*Chapter 10: Lesson 2 Skills Practice ODDS*
4/16	Pythagorean Theorem	*Woot Math: "Square Roots and the Pythagorean Theorem"*	Chapter 10: Lesson 5 Skills Practice ODDS
4/17	Pythagorean Theorem	***Khanacademy: Use***	Chapter 10: Lesson 5

		Pythagorean Theorem to Find Right Triangle Side Lengths	Study Guide and Intervention EVENS both sides
4/20	Ratio of Triangle Side Lengths	***Khanacademy: Trig Ratios in Right Triangles*** ***Video Assigned to help***	*Chapter 10: Lesson 6 Study Guide and Intervention- Evens both sides* ***See video examples I uploaded to help with this lesson***
4/21	Ratio of Triangle Side Lengths		Chapter 10: Lesson 6 Skills Practice ODDS
4/22	Statistics	*Woot Math: "NBA Playoffs! Mean, Median and Inner Quartile Range"*	*Chapter 12: Lesson 4 Comparing Sets of Data SGI EVENS both sides* ***See video examples I uploaded to help with this lesson***
4/23	Permutations and Combinations	***Khanacademy: Permutations and Combinations*** ***Video assigned to help***	*Chapter 12: Lesson 6 Study Guide and Intervention EVENS both sides* ***See video examples I uploaded to help with this lesson***
4/24	Permutations and Combinations	***Khanacademy: Combinations*** ***Video Assigned to help***	*Chapter 12: Lesson 6 Skills ODDS*
4/27	Permutations and Combinations	*Woot Math: "Permutation Practice"*	*Chapter 12: Lesson 6 Word Problem Practice*
4/28	Rational Functions	*Woot Math: "Inverse Variation"*	*Chapter 11: Lesson 1 Study Guide and Intervention EVENS both sides*
4/29	Rational Functions	***Khanacademy: Recognize Direct and Inverse Variation*** ***Video assigned to help***	*Chapter 11: Lesson 1 Study Guide and Intervention EVENS both sides* ***If you did not choose this one yesterday***
4/30	Rational Functions	***Khanacademy: Rational Functions Points of Discontinuity*** *Vertical Asymptote: Where denominator is 0* *Removable Discontinuity (hole in the graph): Where a group was valued at 0*	*Chapter 11: Lesson 2 Study Guide and Intervention EVENS both sides* ***See video examples I uploaded to help with this lesson***

		and was factored and canceled *<u>x-Intercept:</u> where numerator is 0*	
5/1	Rational Functions	*Woot Math: "Finding Asymptotes and Holes"*	*Chapter 11: Lesson 2 Skills Practice ODDS*

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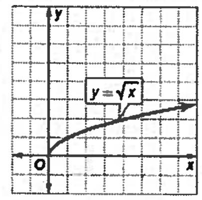
10-1 Study Guide and Intervention

Square Root Functions

Dilations of Radical Functions A **square root function** contains the square root of a variable. Square root functions are a type of **radical function.**

In order for a square root to be a real number, the **radicand,** or the expression under the radical sign, cannot be negative. Values that make the radicand negative are not included in the domain.

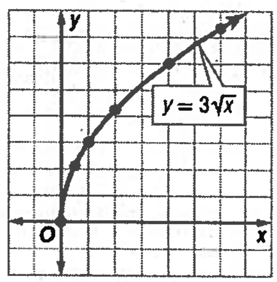
Square Root Function	Parent function: $f(x) = \sqrt{x}$ Type of graph: curve Domain: $\{x \mid x \geq 0\}$ Range: $\{y \mid y \geq 0\}$	

Example: Graph $y = 3\sqrt{x}$. State the domain and range.

Step 1 Make a table. Choose nonnegative values for x

x	y
0	0
0.5	≈ 2.12
1	3
2	≈ 4.24
4	6
6	≈ 7.35

Step 2 Plot points and draw a smooth curve.

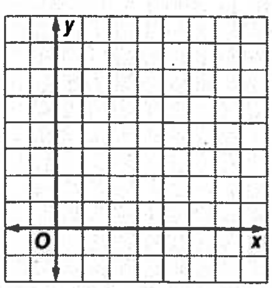


The domain is $\{x \mid x \geq 0\}$ and the range is $\{y \mid y \geq 0\}$.

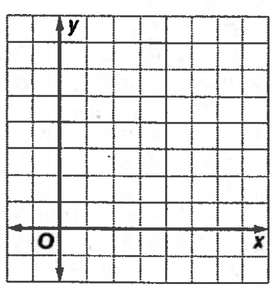
Exercises

Graph each function, and compare to the parent graph. State the domain and range.

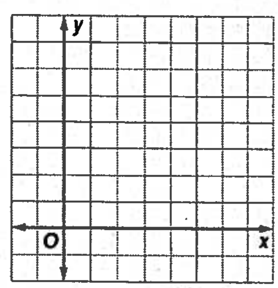
1. $y = \frac{3}{2}\sqrt{x}$



2. $y = 4\sqrt{x}$



3. $y = \frac{5}{2}\sqrt{x}$



10-1 Study Guide and Intervention *(continued)*

Square Root Functions

Reflections and Translations of Radical Functions Radical functions, like quadratic functions, can be translated horizontally and vertically, as well as reflected across the x-axis. To draw the graph of $y = a\sqrt{x + h} + k$, follow these steps.

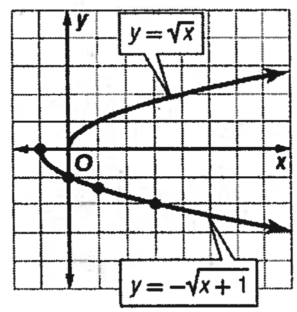
Graphs of Square Root Functions	**Step 1** Draw the graph of $y = a\sqrt{x}$. The graph starts at the origin and passes through the point at $(1, a)$. If $a > 0$, the graph is in the 1st quadrant. If $a < 0$, the graph is reflected across the x-axis and is in the 4th quadrant. **Step 2** Translate the graph $\mid k \mid$ units up if k is positive and down if k is negative. **Step 3** Translate the graph $\mid h \mid$ units left if h is positive and right if h is negative.

Example: Graph $y = -\sqrt{x + 1}$ and compare to the parent graph. State the domain and range.

Step 1 Make a table of values.

x	−1	0	1	3	8
y	0	−1	−1.41	−2	−3

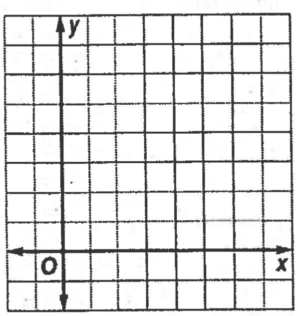
Step 2 This is a horizontal translation 1 unit to the left of the parent function and reflected across the x-axis. The domain is $\{x \mid x \geq -1\}$ and the range is $\{y \mid y \leq 0\}$.



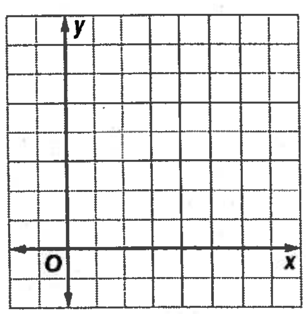
Exercises

Graph each function, and compare to the parent graph. State the domain and range.

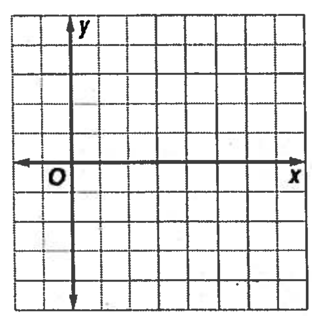
1. $y = \sqrt{x} + 3$



2. $y = \sqrt{x - 1}$



3. $y = -\sqrt{x - 1}$



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10-2 Study Guide and Intervention

Simplifying Radical Expressions

Product Property of Square Roots The **Product Property of Square Roots** and prime factorization can be used to simplify expressions involving irrational square roots. When you simplify radical expressions with variables, use absolute value to ensure nonnegative results.

Product Property of Square Roots	For any numbers a and b, where $a \geq 0$ and $b \geq 0$, $\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$.

Example 1: Simplify $\sqrt{180}$.

$\sqrt{180} = \sqrt{2 \cdot 2 \cdot 3 \cdot 3 \cdot 5}$	Prime factorization of 180
$= \sqrt{2^2} \cdot \sqrt{3^2} \cdot \sqrt{5}$	Product Property of Square Roots
$= 2 \cdot 3 \cdot \sqrt{5}$	Simplify.
$= 6\sqrt{5}$	Simplify.

Example 2: Simplify $\sqrt{120a^2 \cdot b^5 \cdot c^4}$.

$\sqrt{120a^2 \cdot b^5 \cdot c^4}$

$= \sqrt{2^3 \cdot 3 \cdot 5 \cdot a^2 \cdot b^5 \cdot c^4}$

$= \sqrt{2^2} \cdot \sqrt{2} \cdot \sqrt{3} \cdot \sqrt{5} \cdot \sqrt{a^2} \cdot \sqrt{b^4 \cdot b} \cdot \sqrt{c^4}$

$= 2 \cdot \sqrt{2} \cdot \sqrt{3} \cdot \sqrt{5} \cdot |a| \cdot b^2 \cdot \sqrt{b} \cdot c^2$

$= 2|a|b^2c^2\sqrt{30b}$

Exercises

Simplify each expression.

1. $\sqrt{28}$

2. $\sqrt{68}$

3. $\sqrt{60}$

4. $\sqrt{75}$

5. $\sqrt{162}$

6. $\sqrt{3} \cdot \sqrt{6}$

7. $\sqrt{2} \cdot \sqrt{5}$

8. $\sqrt{5} \cdot \sqrt{10}$

9. $\sqrt{4a^2}$

10. $\sqrt{9x^4}$

11. $\sqrt{300a^4}$

12. $\sqrt{128c^6}$

13. $4\sqrt{10} \cdot 3\sqrt{6}$

14. $\sqrt{3x^2} \cdot 3\sqrt{3x^4}$

15. $\sqrt{20a^2b^4}$

16. $\sqrt{100x^3y}$

17. $\sqrt{24a^4b^2}$

18. $\sqrt{81x^4y^2}$

19. $\sqrt{150a^2b^2c}$

20. $\sqrt{72a^6b^3c^2}$

21. $\sqrt{45x^2y^5z^8}$

22. $\sqrt{98x^4y^6z^2}$

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10-2 Study Guide and Intervention *(continued)*

Simplifying Radical Expressions

Quotient Property of Square Roots A fraction containing radicals is in simplest form if no radicals are left in the denominator. The **Quotient Property of Square Roots** and **rationalizing the denominator** can be used to simplify radical expressions that involve division. When you rationalize the denominator, you multiply the numerator and denominator by a radical expression that gives a rational number in the denominator.

Quotient Property of Square Roots	For any numbers a and b, where $a \geq 0$ and $b > 0$, $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$.

Example: Simplify $\sqrt{\frac{56}{45}}$.

$\sqrt{\frac{56}{45}} = \sqrt{\frac{4 \cdot 14}{9 \cdot 5}}$ Factor 56 and 45.

$= \frac{2 \cdot \sqrt{14}}{3 \cdot \sqrt{5}}$ Simplify the numerator and denominator.

$= \frac{2\sqrt{14}}{3\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}}$ Multiply by $\frac{\sqrt{5}}{\sqrt{5}}$ to rationalize the denominator.

$= \frac{2\sqrt{70}}{15}$ Product Property of Square Roots

Exercises

Simplify each expression.

1. $\frac{\sqrt{9}}{\sqrt{18}}$

2. $\frac{\sqrt{8}}{\sqrt{24}}$

3. $\frac{\sqrt{100}}{\sqrt{121}}$

4. $\frac{\sqrt{75}}{\sqrt{3}}$

5. $\frac{8\sqrt{2}}{2\sqrt{8}}$

6. $\sqrt{\frac{2}{5}} \cdot \sqrt{\frac{6}{5}}$

7. $\sqrt{\frac{3}{4}} \cdot \sqrt{\frac{5}{2}}$

8. $\sqrt{\frac{5}{7}} \cdot \sqrt{\frac{2}{5}}$

9. $\sqrt{\frac{3a^2}{10b^6}}$

10. $\sqrt{\frac{x^6}{y^4}}$

11. $\sqrt{\frac{100a^4}{144b^8}}$

12. $\sqrt{\frac{75b^3c^6}{a^2}}$

13. $\frac{\sqrt{4}}{3 - \sqrt{5}}$

14. $\frac{\sqrt{8}}{2 + \sqrt{3}}$

15. $\frac{\sqrt{5}}{5 + \sqrt{5}}$

16. $\frac{\sqrt{8}}{2\sqrt{7} + 4\sqrt{10}}$

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10-2 Skills Practice
Simplifying Radical Expressions

Simplify each expression.

1. $\sqrt{28}$

2. $\sqrt{40}$

3. $\sqrt{72}$

4. $\sqrt{99}$

5. $\sqrt{2} \cdot \sqrt{10}$

6. $\sqrt{5} \cdot \sqrt{60}$

7. $3\sqrt{5} \cdot \sqrt{5}$

8. $\sqrt{6} \cdot 4\sqrt{24}$

9. $2\sqrt{3} \cdot 3\sqrt{15}$

10. $\sqrt{16b^4}$

11. $\sqrt{81a^2d^4}$

12. $\sqrt{40x^4y^6}$

13. $\sqrt{75m^5p^2}$

14. $\sqrt{\frac{5}{3}}$

15. $\sqrt{\frac{1}{6}}$

16. $\sqrt{\frac{6}{7}} \cdot \sqrt{\frac{1}{3}}$

17. $\sqrt{\frac{q}{12}}$

18. $\sqrt{\frac{4h}{5}}$

19. $\sqrt{\frac{12}{b^2}}$

20. $\sqrt{\frac{45}{4m^4}}$

21. $\frac{2}{4+\sqrt{5}}$

22. $\frac{3}{2-\sqrt{3}}$

23. $\frac{5}{7+\sqrt{7}}$

24. $\frac{4}{3-\sqrt{2}}$

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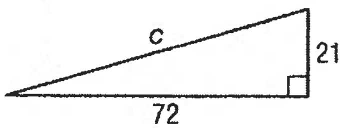
10-5 Skills Practice
The Pythagorean Theorem

Find each missing length. If necessary, round to the nearest hundredth.

1.



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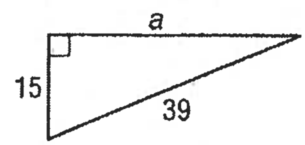

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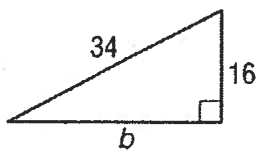

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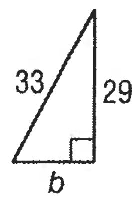

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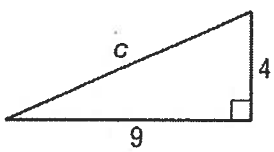

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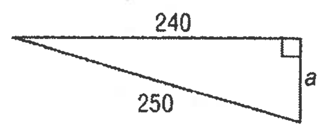

Determine whether each set of measures can be sides of a right triangle. Then determine whether they form a Pythagorean triple.

7. 7, 24, 25

8. 15, 30, 34

9. 16, 28, 32

10. 18, 24, 30

11. 15, 36, 39

12. 5, 7, $\sqrt{74}$

13. 4, 5, 6

14. 10, 11, $\sqrt{221}$

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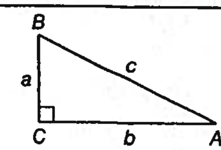
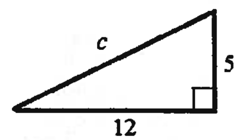
10-5 Study Guide and Intervention

The Pythagorean Theorem

The Pythagorean Theorem The side opposite the right angle in a right triangle is called the **hypotenuse**. This side is always the longest side of a right triangle. The other two sides are called the **legs** of the triangle. To find the length of any side of a right triangle, given the lengths of the other two sides, you can use the **Pythagorean Theorem**.

Pythagorean Theorem	If a and b are the measures of the legs of a right triangle and c is the measure of the hypotenuse, then $c^2 = a^2 + b^2$.	

Example: Find the missing length.



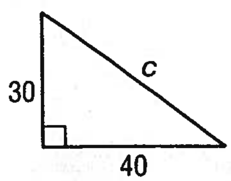
$c^2 = a^2 + b^2$	Pythagorean Theorem
$c^2 = 5^2 + 12^2$	$a = 5$ and $b = 12$
$c^2 = 169$	Simplify.
$c = \sqrt{169}$	Take the square root of each side.
$c = \pm 13$	Simplify.

Because lengths are always nonnegative, the length of the hypotenuse is 13.

Exercises

Find the length of each missing side. If necessary, round to the nearest hundredth.

1.



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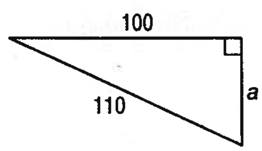

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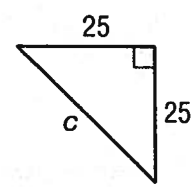

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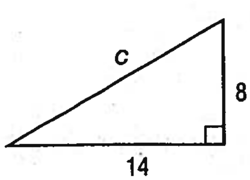

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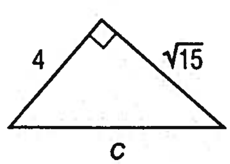

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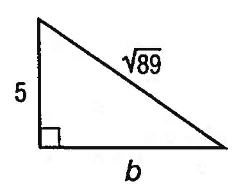

10-5 Study Guide and Intervention *(continued)*

The Pythagorean Theorem

Right Triangles If a and b are the measures of the shorter sides of a triangle, c is the measure of the longest side, and $c^2 = a^2 + b^2$, then the triangle is a right triangle.

Example: Determine whether each set of measures can be sides of a right triangle.

a. 10, 12, 14

Since the greatest measure is 14, let $c = 14$, $a = 10$, and $b = 12$.

$c^2 = a^2 + b^2$	Pythagorean Theorem
$14^2 \stackrel{?}{=} 10^2 + 12^2$	$a = 10, b = 12, c = 14$
$196 \stackrel{?}{=} 100 + 144$	Multiply.
$196 \neq 244$	Add.

Since $c^2 \neq a^2 + b^2$, segments with these measures cannot form a right triangle.

b. 7, 24, 25

Since the greatest measure is 25, let $c = 25$, $a = 7$, and $b = 24$.

$c^2 = a^2 + b^2$	Pythagorean Theorem
$25^2 \stackrel{?}{=} 7^2 + 24^2$	$a = 7, b = 24, c = 25$
$625 \stackrel{?}{=} 49 + 576$	Multiply.
$625 = 625$	Add.

Since $c^2 = a^2 + b^2$, segments with these measures can form a right triangle.

Exercises

Determine whether each set of measures can be sides of a right triangle. Then determine whether they form a Pythagorean triple.

1. 14, 48, 50

2. 6, 8, 10

3. 8, 8, 10

4. 90, 120, 150

5. 15, 20, 25

6. 4, 8, $4\sqrt{5}$

7. 2, 2, $\sqrt{8}$

8. 4, 4, $\sqrt{20}$

9. 25, 30, 35

10. 24, 36, 48

11. 18, 80, 82

12. 150, 200, 250

13. 100, 200, 300

14. 500, 1200, 1300

15. 700, 1000, 1300

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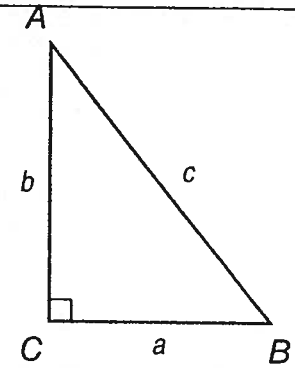
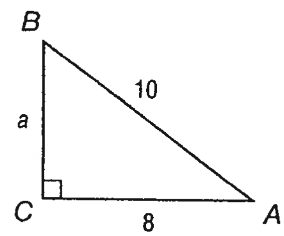
10-6 Study Guide and Intervention
Trigonometric Ratios

Trigonometric Ratios Trigonometry is the study of relationships of the angles and the sides of a right triangle. The three most common trigonometric ratios are the **sine**, **cosine**, and **tangent**.

sine of $\angle A = \frac{\text{leg opposite } \angle A}{\text{hypotenuse}}$ sine of $\angle B = \frac{\text{leg opposite } \angle B}{\text{hypotenus}}$	$\sin A = \frac{a}{c}$ $\sin B = \frac{b}{c}$	
cosine of $\angle A = \frac{\text{leg adjacent to } \angle A}{\text{hypotenus}}$ cosine of $\angle B = \frac{\text{leg adjacent to } \angle B}{\text{hypotenus}}$	$\cos A = \frac{b}{c}$ $\cos B = \frac{a}{c}$	
tangent of $\angle A = \frac{\text{leg opposite } \angle A}{\text{leg adjacent to } \angle A}$ tangent of $\angle B = \frac{\text{leg opposite } \angle B}{\text{leg adjacent to } \angle B}$	$\tan A = \frac{a}{b}$ $\tan B = \frac{b}{a}$	

Example: Find the values of the three trigonometric ratios for angle *A*.

Step 1 Use the Pythagorean Theorem to find *BC*.



$a^2 + b^2 = c^2$	Pythagorean Theorem
$a^2 + 8^2 = 10^2$	$b = 8$ and $c = 10$
$a^2 + 64 = 100$	Simplify.
$a^2 = 36$	Subtract 64 from each side.
$a = 6$	Take the positive square root of each side.

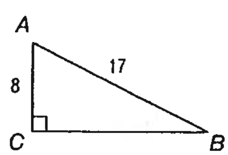
Step 2 Use the side lengths to write the trigonometric ratios.

$\sin A = \frac{\text{opp}}{\text{hyp}} = \frac{6}{10} = \frac{3}{5}$ $\cos A = \frac{\text{adj}}{\text{hyp}} = \frac{8}{10} = \frac{4}{5}$ $\tan A = \frac{\text{opp}}{\text{adj}} = \frac{6}{8} = \frac{3}{4}$

Exercises

Find the values of the three trigonometric ratios for angle *A*.

1.



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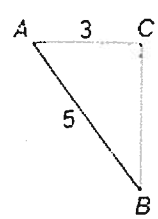

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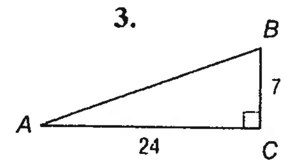


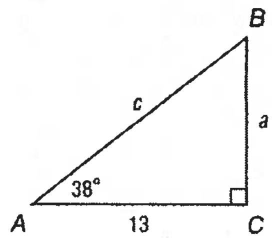
Use a calculator to find the value of each trigonometric ratio to the nearest ten-thousandth.

4. sin 40° **5.** cos 25° **6.** tan 85°

10-6 Study Guide and Intervention *(continued)*
Trigonometric Ratios

Use Trigonometric Ratios When you find all of the unknown measures of the sides and angles of a right triangle, you are **solving the triangle**. You can find the missing measures of a right triangle if you know the measure of two sides of the triangle, or the measure of one side and the measure of one acute angle.

Example: Solve the right triangle. Round each side length to the nearest tenth.



Step 1 Find the measure of $\angle B$. The sum of the measures of the angles in a triangle is 180.

$180° - (90° + 38°) = 52°$

The measure of $\angle B$ is 52°.

Step 2 Find the measure of $\overline{AB}$. Because you are given the measure of the side adjacent to $\angle A$ and are finding the measure of the hypotenuse, use the cosine ratio.

$\cos 38° = \frac{13}{c}$	Definition of cosine
$c \cos 38° = 13$	Multiply each side by c.
$c = \frac{13}{\cos 38°}$	Divide each side by cos 38°.
$c \approx 16.5$	Use a calculator.

So, the measure of $\overline{AB}$ is about 16.5.

Step 3 Find the measure of $\overline{BC}$. Because you are given the measure of the side adjacent to $\angle A$ and are finding the measure of the side opposite $\angle A$, use the tangent ratio.

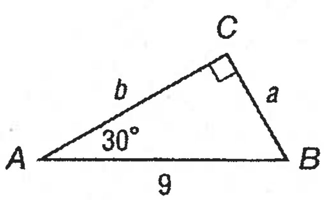
$\tan 38° = \frac{a}{13}$	Definition of tangent
$13 \tan 38° = a$	Multiply each side by 13.
$10.2 \approx a$	Use a calculator.

So, the measure of $\overline{BC}$ is about 10.2.

Exercises

Solve each right triangle. Round each side length to the nearest tenth.

1.



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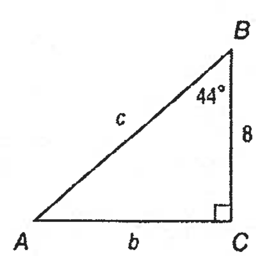

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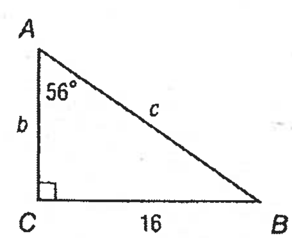

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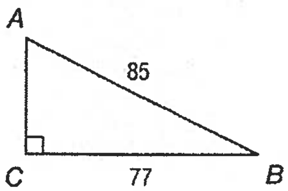
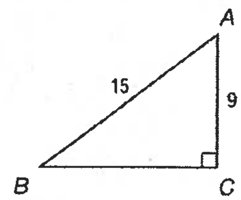
10-6 Skills Practice
Trigonometric Ratio

Find the values of the three trigonometric ratios for angle *A*.

1.



2.



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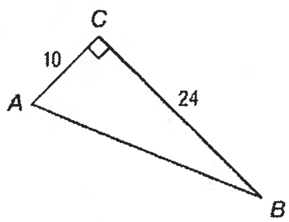

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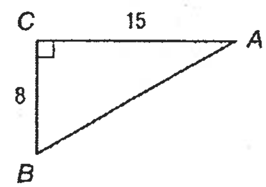


Use a calculator to find the value of each trigonometric ratio to the nearest ten-thousandth.

5. sin 18°

6. cos 68°

7. tan 27°

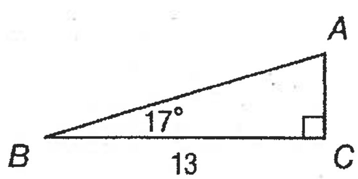
8. cos 60°

9. tan 75°

10. sin 9°

Solve each right triangle. Round each side length to the nearest tenth.

11.



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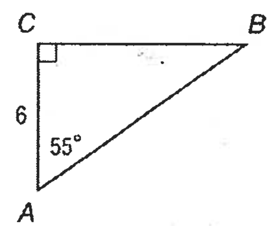
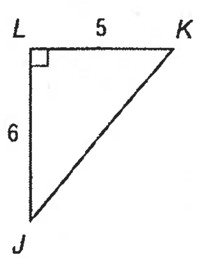


Find $m\angle J$ for each right triangle to the nearest degree.

13.



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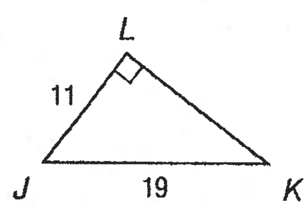

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12-4 Study Guide and Intervention

Comparing Sets of Data

Transformation of Data When an operation is performed on every value of a data set, the statistics for the new set of data can be found using the statistics from the original set of data.

Transformations Using Addition
If a real number k is added to every value in a set of data, then: • the mean, median, and mode of the new data set can be found by adding k to the mean, median, and mode of the original data set, and • the range and standard deviation will not change.

Transformations Using Multiplication
If every value in a set of data is multiplied by a constant k, $k > 0$, then the mean, median, mode, range, and standard deviation of the new data set can be found by multiplying each original statistic by k.

Example 1: Find the mean, median, mode, range, and standard deviation of the data set obtained after adding 9 to each value.

12, 10, 15, 17, 15, 9, 10, 15, 12, 14

The mean, median, mode, range, and standard deviation of the original data set are 12.9, 13, 15, 8, and about 2.5, respectively. Add 9 to the mean, median, and mode. The range and standard deviation are unchanged.

Mean 21.9 Median 22 Mode 24
Range 8 Standard Deviation 2.5

Example 2: Find the mean, median, mode, range, and standard deviation of the data set obtained after multiplying each value by 1.5.

4, 3, 7, 6, 2, 6, 8, 5, 4, 6, 7, 2

The mean, median, mode, range, and standard deviation of the original data set are 5, 5.5, 6, 6, and about 1.9, respectively. Multiply the mean, median, mode, range, and standard deviation by 1.5.

Mean 7.5 Median 8.3 Mode 9
Range 9 Standard Deviation 2.85

Exercises

Find the mean, median, mode, range, and standard deviation of each data set that is obtained after adding the given constant to each value.

1. 33, 38, 29, 35, 27, 34, 36, 28, 41, 26; + 11

2. 8, 9, 3, 6, 12, 9, 3, 16, 9, 11; + (–3)

Find the mean, median, mode, range, and standard deviation of each data set that is obtained after multiplying each value by the given constant.

3. 2, 1, 8, 6, 3, 1, 7, 5, 7, 2, 4, 2; × 4

4. 22, 26, 30, 27, 25, 23, 31, 20; × 0.5

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12-4 Study Guide and Intervention *(continued)*

Comparing Sets of Data

Comparing Distributions When comparing two sets of data, use

- the means and standard deviations if the distributions are both symmetric, or
- the five-number summaries if the distributions are both skewed, or if one distribution is symmetric and the other is skewed.

Example: ATTENDANCE The attendance for each of the PTSA meetings at the two elementary schools in Gahanna Heights School District is shown.

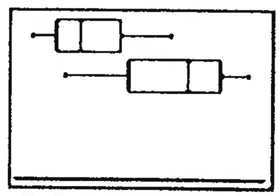
Rocky Run Elementary
20, 56, 25, 45, 41, 27, 28,
51, 30, 34, 23, 37

Trail Woods Elementary
76, 63, 57, 69, 50, 54, 40,
67, 36, 65, 74, 28

a. Use a graphing calculator to construct a box-and-whisker plot for each set of data. Then describe the shape of each distribution.

Enter Rocky Run's attendance as L1 and Trail Woods' as L2. Graph both box-and-whisker plots on the same screen.

For Rocky Run, the distribution is positively skewed. For Trail Woods, the distribution is negatively skewed.



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b. Compare the data sets using either the means and standard deviations or the five-number summaries. Justify your choice.

Both distributions are skewed, so use the five-number summaries to compare the data. The maximum for Rocky Run is 56, while the median for Trail Woods is 60. This means that at half of the meetings at Trail Woods, the attendance was greater than at any of the meetings at Rocky Run. We can conclude that overall, the attendance at Trail Woods' meetings was greater than the attendance at Rocky Run's meetings.

Exercise

SWIMMING Gracie's times in the 50-yard freestyle over two years are shown.

Sophomore Year (seconds)
24.5, 25.7, 24.9, 25.3, 25.8, 25.9, 26.1,
26.3, 26.4, 26.6, 26.9, 27.5

Junior Year (seconds)
24.2, 24.4, 24.5, 24.6, 24.6, 24.9, 25.1,
24.1, 24.1, 23.9, 23.7, 23.5

a. Use a graphing calculator to construct a box-and-whisker plot for each set of data. Then describe the shape of each distribution.

b. Compare the data sets using either the means and standard deviations or the five number summaries. Justify your choice.

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12-6 Study Guide and Intervention

Permutations and Combinations

Permutations An arrangement or listing in which order or placement is important is called a **permutation.** For example, the arrangement AB of choices A and B is different from the arrangement BA of these same two choices.

Permutations	The number of permutations of n objects taken r at a time is $P(n, r) = \frac{n!}{(n-r)!}$.

Example 1: Find $P(6, 2)$.

$P(n, r) = \frac{n!}{(n-r)!}$ Permutation Formula

$P(6, 2) = \frac{6!}{(6-2)!}$ $n = 6$, $r = 2$

$= \frac{6!}{4!}$ Simplify.

$= \frac{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{4 \cdot 3 \cdot 2 \cdot 1}$ Definition of factorial

$= 6 \cdot 5$ or 30 Simplify.

There are 30 permutations of 6 objects taken 2 at a time.

Example 2: PASSWORDS A specific program requires the user to enter a 5-digit password. The digits cannot repeat and can be any five of the digits 1, 2, 3, 4, 7, 8, and 9.

a. How many different passwords are possible?

$P(n, r) = n!(n - r)!$

$P(7, 5) = \frac{7!}{(7-5)!}$

$= \frac{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{2 \cdot 1}$

$= 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3$ or 2520

There are 2520 ways to create a password.

b. What is the probability that the first two digits are odd numbers with the other digits any of the remaining numbers?

$P(\text{first two digits odd}) = \frac{\text{number of favorable outcomes}}{\text{number of possible outcomes}}$

favorable outcomes: There are 4 choices for the first 2 digits and 5 choices for the remaining 3 digits. $P(4, 2) \cdot P(5, 3)$

possible outcomes: There are 7 choices for the 5 digits. $P(7, 5)$

The probability is $\frac{P(4,2) \cdot P(5,3)}{P(7,5)} = \frac{720}{2520}$ or about 28.6%.

Exercises

Evaluate each expression.

1. $P(7, 4)$

2. $P(12, 7)$

3. $P(9, 9)$

4. CLUBS A club with ten members wants to choose a president, vice-president, secretary, and treasurer. Six of the members are women, and four are men.

a. How many different sets of officers are possible?

b. What is the probability that all officers will be women.

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12-6 Study Guide and Intervention *(continued)*

Permutations and Combinations

Combinations An arrangement or listing in which order is not important is called a combination. For example, AB and BA are the same combination of A and B.

Combinations	The number of combinations of n objects taken r at a time is $C(n, r) = \frac{n!}{(n-r)!\,r!}$.

Example: A club with ten members wants to choose a committee of four members. Six of the members are women, and four are men.

a. How many different committees are possible?

$C(n, r) = \frac{n!}{(n-r)!\,r!}$ — Combination Formula

$C(10, 4) = \frac{10!}{(10-4)!\,4!}$ — $n = 10$, $r = 4$

$= \frac{10 \cdot 9 \cdot 8 \cdot 7}{4!}$ — Divide by the GCF 6!.

$= 210$ — Simplify.

There are 210 ways to choose a committee of four when order is not important.

b. If the committee is chosen randomly, what is the probability that two members of the committee are men?

Probability (2 men and 2 women) $= \frac{\text{number of favorable outcomes}}{\text{number of possible outcomes}}$

favorable outcomes: There are 4 choices for the 2 men and 6 choices for the 2 remaining spots. $C(4, 2) \cdot C(6, 2)$

The probability is $\frac{C(4,2) \cdot C(6,2)}{C(10,4)} = \frac{90}{210}$ or about 42.9%.

Exercises

Evaluate each expression.

1. $C(7, 3)$

2. $C(12, 8)$

3. $C(9, 9)$

4. COMMITTEES In how many ways can a club with 9 members choose a two-member sub-committee?

5. BOOK CLUBS A book club offers its members a book each month for a year from a selection of 24 books. Ten of the books are biographies and 14 of the books are fiction.

a. How many ways could the members select 12 books?

b. What is the probability that 5 biographies and 7 fiction books will be chosen?

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12-6 Skills Practice
Permutations and Combinations

Identify each situation as a *permutation* or a *combination*.

1. choosing 16 songs to burn to a CD

2. choosing 3 sports to play at a camp

3. choosing a batting order for kickball

4. choosing outfits to take on vacation

5. choosing where people sit in the car on a trip

6. SCHOOL PLAY Joseph and eight friends are attending the school play. How many ways can Joseph and his friends sit in 9 empty seats?

7. VIDEOS Sanjay is arranging his 6 favorite videos on a shelf. In how many ways can he do this?

8. PASSWORDS Any letter or number can be used for a 4-digit password with no repeating. What is the probability that the first two digits are numbers and the second two digits are letters?

9. MONEY A bag contains 12 one-dollar-bills and 6 hundred-dollar-bills. If 5 bills are randomly selected from the bag, what is the probability that 3 are one-dollar bills and 2 are hundred-dollar bills?

Evaluate each expression.

10. $C(11, 2)$

11. $P(5, 4)$

12. $C(14, 5)$

13. $C(11, 6)$

14. $P(4, 2)$

15. $C(8, 6)$

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12-6 Word Problem Practice
Permutations and Combinations

1. **CHORES** In Ashley's family there are 4 children and her mother and father. If two of the six must help clear the table after dinner, how many ways can two family members be paired?

2. **PUBLIC SERVICE** A county computer randomly selects jurors from the lawyer-approved potential juror list. How many ways are there for 12 jurors to be chosen from a pool of 20?

Potential Jurors	
Fred	Jennifer
Sally	Carla
Mike	Sam
Tom	Sylvia
Maria	Jamal
Ted	Greg
Carlos	Todd
Dorothy	Lauren
Irene	Maya
Trish	Luke

3. **SPORTS** Hans' basketball team decides to choose two captains each week so that many players get the chance to be captain. Each week, each of the 11 players writes his name on a slip of paper. The papers are then placed in a container and mixed. The last week's captains draw two slips of paper from the container; these two people are captains for the following week. How many different pairs of captains can be formed?

4. **PUZZLES** A popular newspaper puzzle involves a series of letters that can be rearranged to form a word. Will is writing his own version of the game for a school project. He wants to scramble the following word for his puzzle.

numbers

How many ways are there to arrange the letters with the letter *b* as the first letter?

5. **HORSE RACING** There are 15 contenders in a horse race.

   a. How many different ways can the horses finish the race?

   b. How many different ways can horses place first, second, and third?

   c. If all 15 horses have an equal chance of winning and 6 of the horses are female, what is the probability that a female horse places first, second, and third?

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11-1 Study Guide and Intervention

Inverse Variation

Identify and Use Inverse Variations An **inverse variation** is an equation in the form of $y = \frac{k}{x}$ or $xy = k$. If two points (x_1, y_1) and (x_2, y_2) are solutions of an inverse variation, then $x_1 \cdot y_1 = k$ and $x_2 \cdot y_2 = k$.

Product Rule for Inverse Variation	$x_1 \cdot y_1 = x_2 \cdot y_2$

From the product rule, you can form the proportion $\frac{x_1}{x_2} = \frac{y_2}{y_1}$.

Example: If y varies inversely as x and $y = 12$ when $x = 4$, find x when $y = 18$.

Method 1 Use the product rule.

$x_1 \cdot y_1 = x_2 \cdot y_2$	Product rule for inverse variation
$4 \cdot 12 = x_2 \cdot 18$	$x_1 = 4, y_1 = 12, y_2 = 18$
$\frac{48}{18} = x_2$	Divide each side by 18.
$\frac{8}{3} = x_2$	Simplify.

Method 2 Use a proportion.

$\frac{x_1}{x_2} = \frac{y_2}{y_1}$	Proportion for inverse variation
$\frac{4}{x_2} = \frac{18}{12}$	$x_1 = 4, y_1 = 12, y_2 = 18$
$48 = 18x_2$	Cross multiply.
$\frac{8}{3} = x_2$	Simplify.

Both methods show that $x_2 = \frac{8}{3}$ when $y = 18$.

Exercises

Determine whether each table or equation represents an *inverse* or a *direct* variation. Explain.

1.

x	y
3	6
5	10
8	16
12	24

2. $y = 6x$

3. $xy = 15$

Assume that y varies inversely as x. Write an inverse variation equation that relates x and y. Then solve.

4. If $y = 10$ when $x = 5$, find y when $x = 2$.

5. If $y = 8$ when $x = -2$, find y when $x = 4$.

6. If $y = 100$ when $x = 120$, find x when $y = 20$.

7. If $y = -16$ when $x = 4$, find x when $y = 32$.

8. If $y = -7.5$ when $x = 25$, find y when $x = 5$.

9. DRIVING The Gerardi family can travel to Oshkosh, Wisconsin, from Chicago, Illinois, in 4 hours if they drive an average of 45 miles per hour. How long would it take them if they increased their average speed to 50 miles per hour?

10. GEOMETRY For a rectangle with given area, the width of the rectangle varies inversely as the length. If the width of the rectangle is 40 meters when the length is 5 meters, find the width of the rectangle when the length is 20 meters.

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11-1 Study Guide and Intervention *(continued)*

Inverse Variation

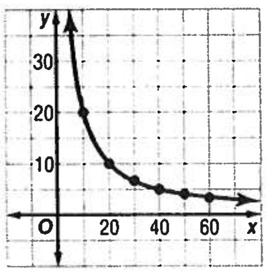
Graph Inverse Variations Situations in which the values of y decrease as the values of x increase are examples of **inverse variation.** We say that y varies inversely as x, or y is inversely proportional to x.

Inverse Variation Equation	an equation of the form $xy = k$, where $k \neq 0$

Example 1: Suppose you drive 200 miles without stopping. The time it takes to travel a distance varies inversely as the rate at which you travel. Let x = speed in miles per hour and y = time in hours. Graph the variation.

The equation $xy = 200$ can be used to represent the situation. Use various speeds to make a table.

x	y
10	20
20	10
30	6.7
40	5
50	4
60	3.3



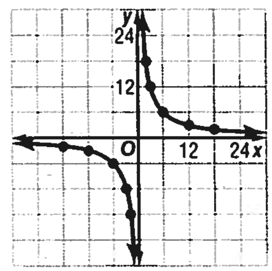
Example 2: Graph an inverse variation in which y varies inversely as x and $y = 3$ when $x = 12$.

Solve for k.

$xy = k$	Inverse variation equation
$12(3) = k$	$x = 12$ and $y = 3$
$36 = k$	Simplify.

Choose values for x and y, which have a product of 36.

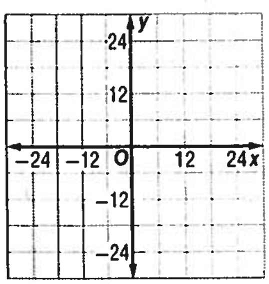
x	y
–6	–6
–3	–12
–2	–18
2	18
3	12
6	6



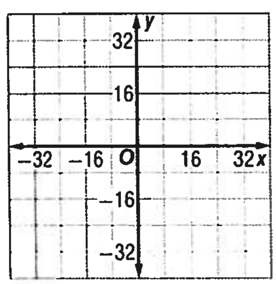
Exercises

Graph each variation if y varies inversely as x.

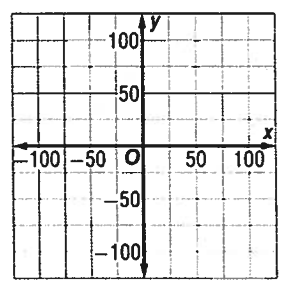
1. $y = 9$ when $x = -3$



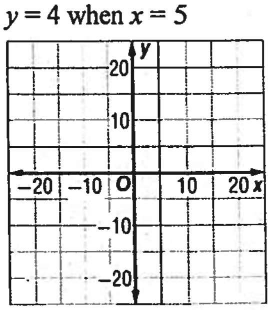
2. $y = 12$ when $x = 4$



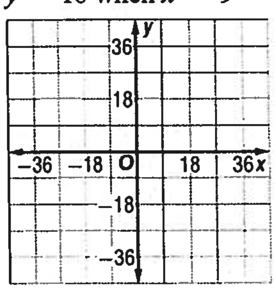
3. $y = -25$ when $x = 5$



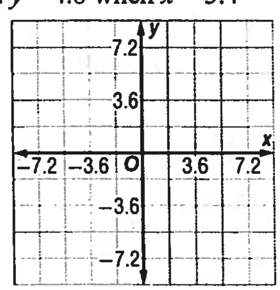
4. $y = 4$ when $x = 5$



5. $y = -18$ when $x = -9$



6. $y = 4.8$ when $x = 5.4$



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11-2 Study Guide and Intervention

Rational Functions

Identify Excluded Values The function $y = \frac{10}{x}$ is an example of a **rational function.** Because division by zero is undefined, any value of a variable that results in a denominator of zero must be excluded from the domain of that variable. These are called **excluded values** of the rational function.

Example: State the excluded value for each function.

a. $y = \frac{3}{x}$

The denominator cannot equal zero. The excluded value is $x = 0$.

b. $y = \frac{4}{x-5}$

$x - 5 = 0$ Set the denominator equal to 0.

$x = 5$ Add 5 to each side.

The excluded value is $x = 5$.

Exercises

State the excluded value for each function.

1. $y = \frac{2}{x}$

2. $y = \frac{1}{x-4}$

3. $y = \frac{x-3}{x+1}$

4. $y = \frac{4}{x-2}$

5. $y = \frac{x}{2x-4}$

6. $y = -\frac{5}{3x}$

7. $y = \frac{3x-2}{x+3}$

8. $y = \frac{x-1}{5x+10}$

9. $y = \frac{x+1}{x}$

10. $y = \frac{x-7}{2x+8}$

11. $y = \frac{x-5}{6x}$

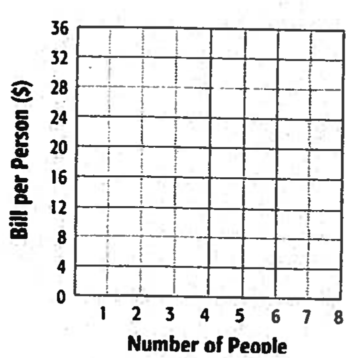
12. $y = \frac{x-2}{x+11}$

13. $y = \frac{7}{3x+21}$

14. $y = \frac{3x-4}{x+4}$

15. $y = \frac{x}{7x-35}$

16. DINING Mya and her friends are eating at a restaurant. The total bill of \$36 is split among x friends. The amount each person pays y is given by $y = \frac{36}{x}$, where x is the number of people. Graph the function.



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11-2 Study Guide and Intervention *(continued)*

Rational Functions

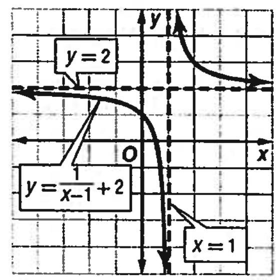
Identify and Use Asymptotes Because excluded vales are undefined, they affect the graph of the function. An **asymptote** is a line that the graph of a function approaches. A rational function in the form $y = \frac{a}{x-b} + c$ has a vertical asymptote at the x-value that makes the denominator equal zero, $x = b$. It has a horizontal asymptote at $y = c$.

Example: Identify the asymptotes of $y = \frac{1}{x-1} + 2$. Then graph the function.

Step 1 Identify and graph the asymptotes using dashed lines.
vertical asymptote: $x = 1$
horizontal asymptote: $y = 2$

Step 2 Make a table of values and plot the points.
Then connect them.

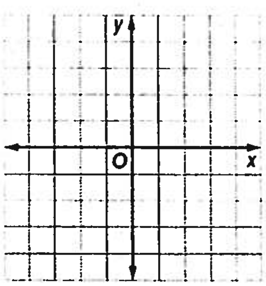
x	–1	0	2	3
y	1.5	1	3	2.5



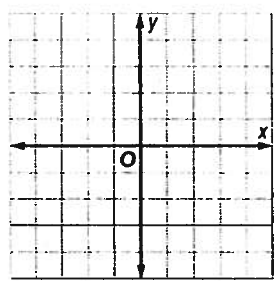
Exercises

Identify the asymptotes of each function. Then graph the function.

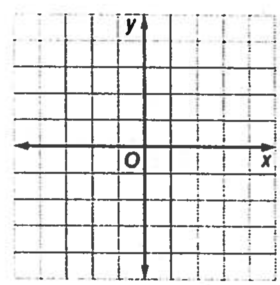
1. $y = \frac{3}{x}$



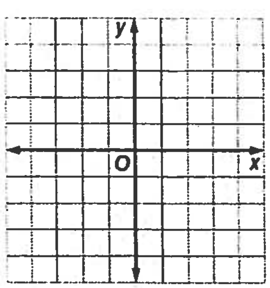
2. $y = \frac{-2}{x}$



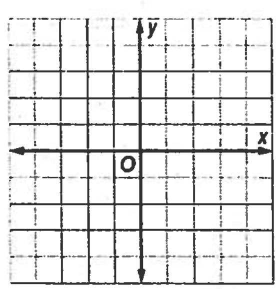
3. $y = \frac{4}{x} + 1$



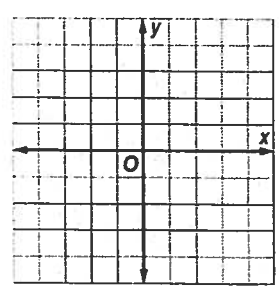
4. $y = \frac{2}{x} - 3$



5. $y = \frac{2}{x+1}$



6. $y = \frac{-2}{x-3}$



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11-2 Skills Practice
Rational Functions

State the excluded value for each function.

1. $y = \frac{6}{x}$

2. $y = \frac{2}{x - 2}$

3. $y = \frac{x}{x + 6}$

4. $y = \frac{x - 3}{x + 4}$

5. $y = \frac{3x - 5}{x + 8}$

6. $y = \frac{-5}{2x - 14}$

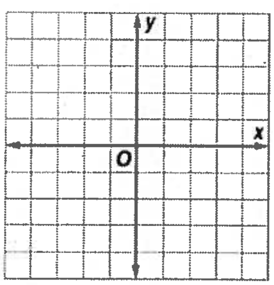
7. $y = \frac{x}{3x + 21}$

8. $y = \frac{x - 1}{9x - 36}$

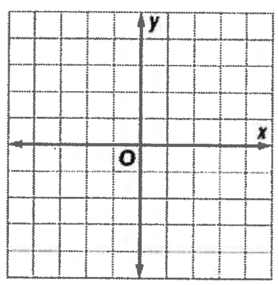
9. $y = \frac{9}{5x + 40}$

Identify the asymptotes of each function. Then graph the function.

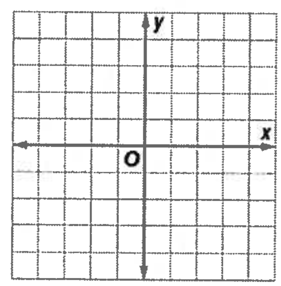
10. $y = \frac{1}{x}$



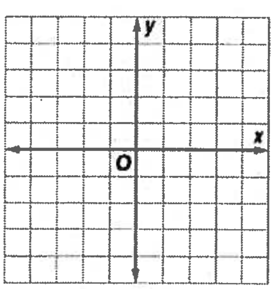
11. $y = \frac{3}{x}$



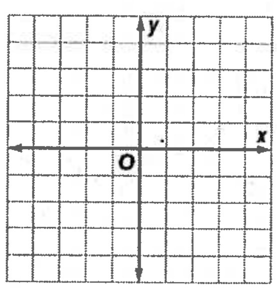
12. $y = \frac{2}{x + 1}$



13. $y = \frac{3}{x - 2}$



14. $y = \frac{2}{x + 1} - 1$



15. $y = \frac{1}{x - 2} + 3$